

# Forcing as a Part of Set-Theoretic Practice

July 1, 2018

In mathematics, statements can be proven to be true, or they can be refuted and then are proven to be false. In set theory however, statements can be proven to be neither provable nor refutable. This fact is called the set-theoretic independence phenomenon, which gives rise to the philosophical independence problem: *What is the status of a statement which is neither provable nor refutable?*

The most powerful method for such independence proofs is the method of forcing. Since more than fifty years, forcing is a part of set-theoretic practice. Much set-theoretic knowledge of today is based on forcing, and the independence problem—the main problem in the philosophy of set theory—is as severe as the forcing method is successful in application.

In order to approach the philosophical independence problem, the role of forcing in set-theoretic practice is investigated. As a suitable method for this descriptive investigation, the analysis of several interviews with expert set theorists is chosen.

Different aspects of the role of forcing are examined:

**Spread of forcing in the set-theoretic community** It is asked to what extent today's set-theoretic practice is determined by forcing. There are contrasting hypotheses on that question. One could have the view that set theory today is actually the theory of the models of set theory, in which the forcing technique is a key method. However, this view is challenged by the fact that there are set theorists who never use forcing. An appropriate description of the spread of forcing in set theory is given.

**Naturalness of forcing at its introduction and today** The following hypothesis is considered: Forcing was unnatural at its introduction, and today it is a natural part of set-theoretic practice. The introduction of forcing in set theory by P. Cohen is mostly considered a surprising event.

But one could also have the view that in the 60's, the time was ripe for the forcing result on AC and CH. The question of the naturalness of forcing today is connected to the first topic. Set theorists who use forcing regularly might find forcing very natural, while set theorists who never use forcing might not agree on that judgement. A refined evaluation of the naturalness of forcing is given.

**Compatibility of the use of forcing and the universe view** The use of forcing might better support a multiverse view than a universe view. For Hamkins, the widespread use of forcing is one essential argument for a multiverse view. But there are also set theorists with a universe view who use forcing. Different possibilities how the use of forcing can be compatible with the universe view are presented.

**Acceptability and use of forcing axioms** The question how forcing axioms are used, and which of their properties are important in connection to the question of acceptability is the main question for this topic. Large cardinal axioms seem to be better candidates for acceptability than forcing axioms. But there are set theorists who say that forcing axioms are natural because they correspond to the idea of a forcing-saturated universe. One could also say that the property of consistency strength weighs much more with regard to acceptability than the difference between large cardinal axioms and forcing axioms. Different views on forcing axioms, and kinds of using them are presented, compared to the case of large cardinal axioms and evaluated with regard to the question of acceptability.