**Ernst Cassirer's Transcendental Account of Mathematical Reasoning**

Cassirer's philosophical agenda revolved around what appears to be a paradoxical goal, that is, to reconcile the Kantian explanation of the possibility of knowledge with the conceptual changes of nineteenth and early twentieth-century science. This paper offers a new discussion of one way in which this paradox manifests itself in Cassirer's philosophy of mathematics. Beginning in 1910, Cassirer articulated a unitary perspective on mathematics as an investigation of structures independently of the nature of individual objects making up those structures. However, a tension remains between Cassirer's demand for the unity of knowledge and his reliance on the structural methods of nineteenth-century mathematics. Cassirer tried to resolve this tension in his early works by pointing out that the loss of unity with regard to the subject-matter of modern mathematics – insofar as this ceases to define itself as the science of numbers and quantities – is compensated by the deeper unity of its method. However, after the development of modern axiomatics, Cassirer realized ever more clearly that mathematics (including the most abstract parts of it) raises new problems of its own. In general, beginning in the 1920s, he acknowledged different types of objectivity at stake in the different ways to understand the world, which he called "symbolic forms." Limiting the consideration to epistemology, it seems that in order to account for the unity of mathematics in the latter sense, it would be inevitable to call into question the unity of knowledge in Cassirer's original account.

More recent discussions of Cassirer's philosophy of mathematics reflect the same tension. Jeremy Heis suggests that a charitable way to read Cassirer today would have to offer a unitary account of mathematical objectivity. By contrast, Thomas Mormann maintains that the central thesis of Cassirer's philosophy from 1910 to his later works is that mathematical and physical knowledge are of the same kind (*sameness thesis*). In order to spell out what the sameness thesis entails, Mormann offers a series of examples of how the extension of both kinds of knowledge requires the introduction of ideal elements. It follows that a consistent development of the sameness thesis in the light of twentieth-century mathematics would have to acknowledge incompatible idealizations. In other words, quite contrary to Heis, Mormann's suggestion is to allow for a plurality of conceptual frameworks in the philosophy of mathematics in order to retain the main insight of the sameness thesis.

This paper aims to clarify how both aspects of Cassirer's philosophy stand together by drawing attention to the transcendental argument at stake with the sameness thesis. By transcendental here I mean all kinds of arguments that set conditions for the possibility of knowledge. In particular, the argument under consideration reflects the structure of a transcendental deduction in Kant's sense: in order to justify the possibility of knowledge, Kant offers a proof that the fundamental concepts of the understanding necessarily apply to the manifold of intuition. According to Cassirer, the logic at work in the formation of numerical concepts necessarily applies to spatial concepts and spatiotemporal relations. This offers an explanation of the extensibility of mathematical knowledge from abstract to empirical domains.

I will contend that Cassirer's argument derives from the reading of the Kantian theory of space articulated by Cassirer's teachers, Hermann Cohen and Paul Natorp, and further developed by Cassirer in the second volume of *The Problem of Knowledge in Modern Philosophy and Science* (1907). I will then turn to how Cassirer connects the account of mathematical reasoning that emerges from this reading to the structuralist methodology of nineteenth-century. My suggestion is that a more careful consideration of the key examples for Cassirer's account can shed light on his long-term strategy to resolve the tension between his emphasis on the unity of mathematics and the sameness thesis. Mathematical and structural reasoning typically include the embedding of a particular domain into a larger structure. Paradigmatic examples of this are the introduction of irrational numbers as limits of converging series of rationals and the generalization of the Euclidean plane to the projective plane. While these examples underpin a unitary perspective on specific mathematical disciplines, I will contend that Cassirer emphasized a no less essential aspect of mathematical concept formation, that is the transposition of structural methods from one specific domain to another. Three examples are particularly relevant here: (1) Richard Dedekind's definition of natural numbers, (2) Felix Klein's use of transfer principles, (3) the construction of a numerical scale on the projective line. These are examples of how structural procedures are transferred across algebraic, numerical and geometrical domains. At the same time, they lend plausibility to Cassirer's argument about the extensibility of such procedures to empirical domains in a unitary but internally articulated view of knowledge.

My suggestion is that Cassirer offers a philosophical account of cases where structural reasoning finds unexpected applications beyond the original ground for its development.